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Optimal Investment Policy for Pension Annuitants in Malaysia under Economic Exposure

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ABSTRACT

This research has conducted a profound evaluation into the annuity market in Malaysia from both the business and individual perspective. To a great extent, analysis is made on how the annuity insurers could better maintain the solvency level of annuity fund which is exposed under significant economic risks by obtaining an optimal fund allocation to maximise investment returns through the development of Asset-Liability Management (ALM). By adapting the concept from Wilkie model, six fundamental models including inflation rate, real interest rate, KLCI stock return, KLCI stock dividend yield, 10-year MGS return as well as Malaysian property return were formulated accordingly to the Box-Jenkins modelling approach. It concludes that the optimal weightages combination suggests that the annuity insurers should allocate 94% of the single premium such as RM100,000 which collected from the annuitant to bond market, whereas the other 5% of the fund should be invested in stock but only 1% of the premium should be allocated into the property investment.

1. Introduction

While many researchers and policy analysts have consistently placed tremendous attention on how retirees should accumulate their wealth through saving and investing in the previous years, many of them are becoming increasingly aware that asset accumulation alone would not fully secure the retirement of an individual without knowing how to convert the accumulated wealth into a retirement consumption stream (Brown, 2004). Indeed, the critical issue of aging populations in many developed countries has raised a substantial interest in the way individuals finance their retirement consumption to deal with the uncertain length of lifespan. For example, in

1950, averagely, a 65-year-old man is expected to live until 78 years old while a woman who aged 65 years old is expected to live until 80 years old. However, today, a man with the same age could live until 82 years old while a same-aged woman could live until 85 years old (Vernon, 2014). Like many other countries, Malaysia is also experiencing a rapid growth of an aging population. For example, according to the World Bank 2016, it shows that the population who ages 65 and above are constantly increasing in the past five years in Malaysia. With such significant improvement in life expectancy of people globally over the last decade, it appears that an individual may face a greater risk of having reduced standard of living during advanced ages if no appropriate financial planning is done before retirement.

From here, it comes in the life annuity, an insurance vehicle that offers a steady stream of periodic incomes for insured individuals as long as they are alive. The main highlight of a life annuity product is that it provides an approach to spread the accumulated wealth of insured individuals over their uncertain retirement period in order to protect them against the risk of outliving their accumulated wealth given uncertainty about longevity (Mitchell, Poterba and Warshawsky, 1997). Even though the considerable longevity risk during old ages has demonstrated the remarkable role of annuities in the portfolios for the elderly, the actual market for individual life annuities is still thin and unpopular (Lockwood, 2012). In contrast with those theoretical outcomes, voluntary annuitisation is almost absent in most of the countries, even the worlds' biggest annuity market in the United Kingdom continues to be unappealing (Rusconi, 2008). Given the significant disconnect between the theory and practice, it is natural to question why the annuity market is so small despite the large welfare gains generated from annuities.

Thus, a number of research are expected to appear and examine the true value of annuities in contributing to optimise the retirees' income in trying to explain the phenomena. However, in reality, most of the previous empirical researchers only establish the life annuities market in developed countries (Shu, Zeithammer and Payne, 2016). Therefore, unlike the previous empirical researchers, this will be the first paper to investigate and deeply expose the annuitisation puzzle in underdeveloped market-Malaysia, given that the annuity market is becoming crucially important nowadays whereby it plays a remarkable part in consumer accumulation of their retirement wealth.

Despite the importance of annuity market in giving better protection to retirees from outliving their retirement income, it is of concern, to identify the ability of annuity insurers in ensuring the sustainability of assets in matching with the outgoing guaranteed income paid to the annuitants. Typically, Asset-Liability Model (ALM) is the most common quantitative approach that being developed in almost every study that aims to discover the optimal strategic asset allocation of pension fund that matches the structure of its liabilities (Wane, 2011). In fact, this model is widely used in numerous papers study on the solvency level of pension schemes in developed countries especially in United States and United Kingdom, but only a limited numbers of researchers are found constructing a simplified ALM for Malaysian annuity fund. In fact, it is surprised that many papers only revealed the theoretical formulas on what need to be done to build ALM instead of demonstrating numerically on the proper development of a complete stochastic asset model as well as liability cash flows for an Asian pension fund (Chong, 2007). Yet, most of the past researchers only concerned on matching the liability of the fund with fixed income assets and equities. Thus, despite the gaps found in the previous studies, this paper is believed to have filled the gaps up by demonstrating steps by steps on each development of asset model and the matching of its liability cash flow with not only the stock and bond return, but also the property returns in Malaysian annuity fund from business perspective.

Relatively from business perspective, the importance of Asset-Liability Management (ALM) to annuity insurers' results from life insurance being principally a liability driven business with assets invested to match with (Gilbert, 2016). Specially, since the life annuity insurers with long liability durations could further be exposed to significant economic risk, it is definitely important to make sure that the annuity insurers take into account the cost of risk when planning for optimal investment strategies. By maximising its fund allocation in ventures that create the most value, it assures the sustainability of insurers in giving continuous protections to retirees. Despite its importance, asset and liability models would be designed to access the risk management relating to the assets and liabilities of Malaysia's annuity insurers in this paper. On the other hand, from the individual perspective, asset-liability modelling would demonstrate a clearer picture on the true value of life annuity towards their retirement. An optimal fund allocation would be defined in this research for individuals to maximise their retirement income by investing in the combination of both non-annuitized assets and annuity products. Therefore, unlike other empirical researchers which only examine the ALM from business perspective, this paper is the first to include the ALM from individual perspective along with the ALM from insurer perspective in Malaysia.

The remainder of this paper is structured as follows: Section 2 discloses an overview of the methodology employed for this review purpose; Section 3 reveals the analysis and discussion made on collected papers based on the factors relevant to each of them under each aspect of Big Data applications; Section 4 presents the research gaps, challenges and limitations found in existing studies to provide suggestions for future developments; Section 5 eventually concludes and suggests future research directions.

2. Methodology

2.1. Research Framework

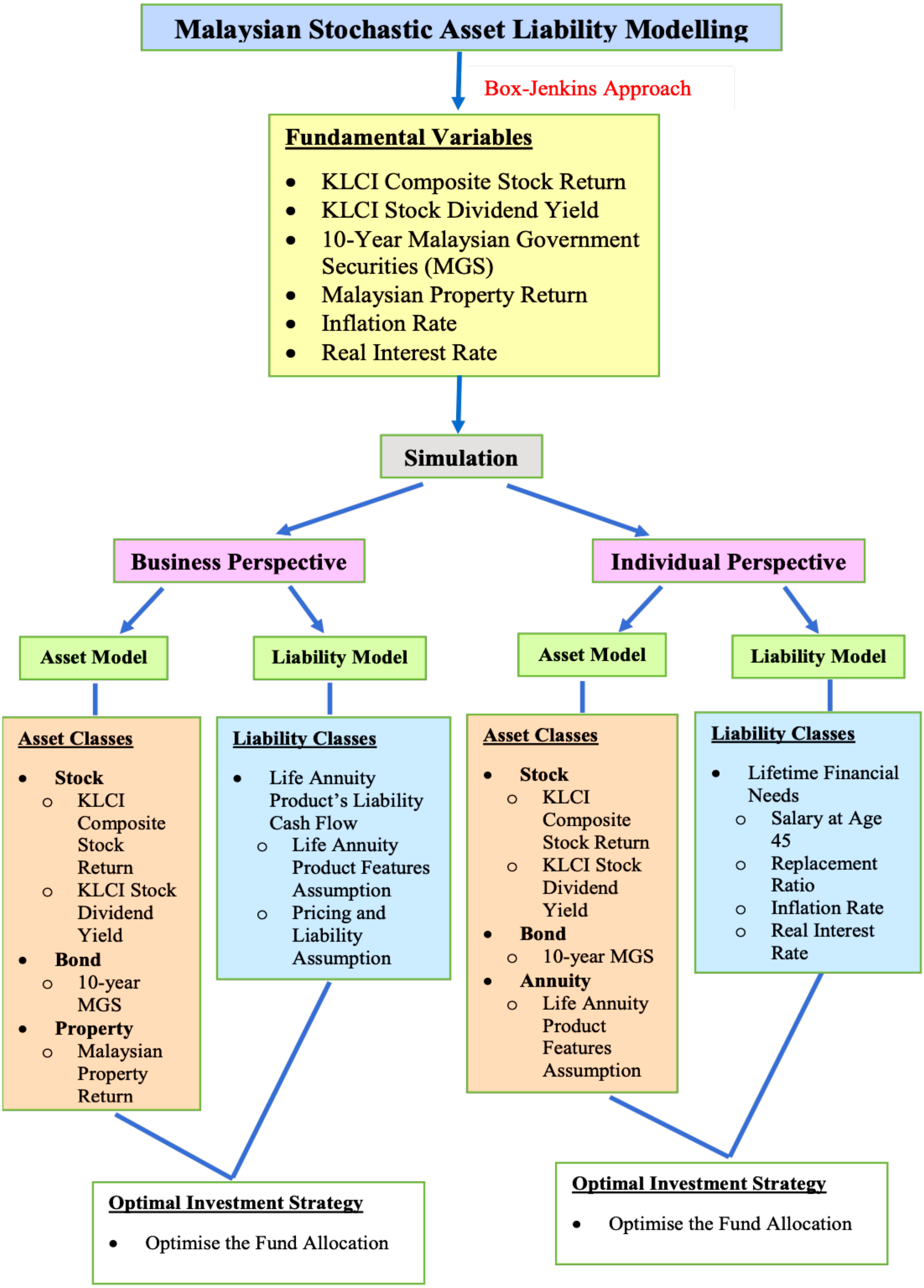


Figure 1: Theoretical Framework of This Research

The above diagram is the theoretical framework constructed for this research which clearly demonstrates the entire process of this research study as well as providing the general framework for data analysis used in this paper. In order to establish ALM from business perspective, the researcher of this paper will firstly construct the asset models for bond, property and equity to obtain the final system of the asset models for running 10000 simulations for 35 years on constructing the distribution for each asset class. At the same time, the liabilities of annuity insurers which associated with mortality risk are modelled through the projection of liability cash flows based on the product features assumption as well as pricing and liability assumption. Consequently, a set of weightages combinations for stock, bond and property in which the sum of each combination must equal to 1 is randomly generated using R. Within this context, the projected liability cash flows are combined with the asset cash flows to obtain the probability whereby the asset cash flows are more than the liability cash flows by substituting each weightages combination into the asset cash flows. From here, an optimal asset allocation which maximises the investment return of annuity insurers could be obtained from the weightages combination that gives the highest probability of solvency. Perhaps, by applying the same optimal weightages combination, a minimum premium required to reach the 90% probability of solvency is obtained.

On the other hand, in order to establish ALM from retirees' perspective, the researcher of this paper will construct the asset models for both asset classes – an annuity product and non-annuity assets which include equity and bond. As this research is aimed to identify the optimal fund allocation for an individual to maximise his assets so that it could cover his lifetime financial needs when he retires at the age of 60, this paper is concerned in accumulating the assets' returns until the age of 60 to demonstrate the total assets' returns which available for consumption in the rest of the life after retiring at 60 years old. Since the stock and bond return models have already been constructed and simulated from business perspective, the simulated results are used to do the accumulation. Apart from that, inflation model and real interest rate model would also be developed to run for simulation as their simulated results are required to be used in formulating the lifetime consumption cash flows. As such, the lifetime consumption cash flows are constructed by taking into account the inflation risk and being discounted back to the age of 60 to get the present value of total lifetime consumption at 60 years of age. Hence, the total accumulated asset cash flows at 60 years old are matched with the total discounted lifetime consumptions at the age of 60 in order to acquire the probability in which the net cash flows are equal or more than zero by substituting each weightages combination along with estimated fund available for investments. By doing so, it gives a list of optimal weightages combinations in maximising the assets' returns for a range of fund available. From here, it allows this research to demonstrate an optimal fund allocation strategy for individuals with different fund available for investments at aged 45 within a range whereby it promises at least 90% of solvency.

2.2 Elaboration of Data Analysis & Data Interpretation

2.2.1 Asset-liability Modelling

Asset-Liability modelling is a quantitative approach that model the interaction and allocation of assets to meet liabilities over time while asset liability management manages the strategy and structure of assets to meet liabilities (Sherris, 2006). This model is widely used by financial institutions to manage their assets and liabilities in order to achieve their business objectives. Consequently, it is also being used to determine the structure of portfolio required to meet annuitants' personal financial needs. Basically, the management of personal financial needs is different from the management of institutional or corporate assets and liabilities. Pension fund shortfalls can be made up by increased funding but not the individuals who are in retirement. Particularly, the different focus on short and long-term needs has focused the pension fund planning on long term deficits, whereas the individuals are required to emphasise on short-term deficits given that the long-term financial safety can be placed at risk by unsuitable short-term structure and assumptions.

Since this research is conducted to examine the annuity market from both insurers' and annuitants' perspective, the primary objective of constructing asset-liability models in this paper is to determine the portfolio structure needed to balance short and long-term financial assets against short and long-term financial needs in order to meet both perspectives. A stochastic asset and liability model which would be adopted in this research consists of three integral parts: an economic and asset projection model; a long-term liability projection model; and a decision-making model used to define an optimal strategy based on the projections of assets and liabilities (Yakoubov, Teeger and Duval, 1999a).

2.2.2 Wilkie Stochastic Investment Model: The Basics in Discrete Time Model

In general, stochastic investment model is one of the asset models which attempts to forecast the variations of prices and returns on different assets or asset classes such as bonds and stocks over time (Yakoubov, Teeger and Duval, 1999). It is always being used for actuarial work and financial planning to compute optimal asset allocation. Specifically, stochastic investment model can be classified into single-asset and multi-asset models. Since the primary objective of this paper is to discover an optimal fund allocation for insurers to maximise return from investing in multiple assets, Wilkie Stochastic Investment Model is chosen to be adopted in this research in conjunction to construct a new model with Malaysia's datasets. In fact, since Wilkie Stochastic Investment Model has been used extensively in asset and liability modelling for pension funds and life insurance in most of the research papers, it is strongly believed that this model is most suitable to be adopted in this paper whereby it shares the similar intention and therefore could be easily adopted to set up the fundamental variables for Malaysian asset models (Yakoubov, Teeger and Duval, 1999).

The Wilkie stochastic investment model, developed by A. D. Wilkie is deeply defined in two papers: the original Wikie model is described in 'A Stochastic Investment Model for Actuarial Use' (Wilkie, 1986) and then being revised, updated and extended in 'More on a Stochastic Asset Model for Actuarial Use' (Wilkie, 1995). The original version of Wilkie model (1986) was developed from UK data over the period between 1919 to 1982 and was made up of four interconnected models for price inflation, share dividend yields, bond dividend yields and long-term interest rates. (Wilkie, 1995) modernised the original model and extended it to comprise an alternative autoregressive conditional heteroscedastic (ARCH) model for price inflation and models for wage inflation, short-term interest rates, property yields and index-linked yields. Moreover, these models were appropriate to fit the data from several developed countries and an exchange rate model was proposed.

Goodwin et al. (2014) describes the Wilkie model as a multivariate model which projects several related economic series together. This is very useful for applications that require consistent projections such as stock prices and inflation rates or fixed interest yields. It is designed for long-term actuarial applications such as simulating assets of financial institutions over many years in the future to study the risk of insolvency. Therefore, this model is very suitable to be the guideline in constructing a new model for this research as this paper aims to determine a long-term strategic allocation of assets for insurers and pension funds in Malaysia (Wilkie, 2009). The Wilkie model is relatively straightforward to implement and has been widely used by UK-based actuaries over the past two decades and has set a benchmark against which any other proposed approach needs to be judged. There are three fundamental inputs to the design of any stochastic model: economic theory, investment practice and historical data (Sahin, 2010). In this research, fundamental inputs such as composite stock return, stock dividend yield, 10-year bond return as well as property return would be involved in formulating each stochastic asset model for assessing the capital management of Malaysia's annuity insurers by adopting the stochastic investment model built upon the work done by Wilkie (1986). On the other hand, from individual perspective, the fundamental inputs which include composite stock return, stock dividend yield, 10-year bond yield, annuity return, real interest rate as well as inflation rate would be used in constructing each stochastic asset model in this research.

2.2.3 Malaysian Stochastic Asset Liability Modelling

Apparently, the same concept and formulas derived from Wilkie model would be applied to the Malaysian investment data collected in the process of formulating each asset class with numerous amendments. Instead of just simply fitting the original Wilkie model to Malaysian data for simulation, adjustments are necessary to be made as (Ishak, 2015) has argued that Wilkie model may not seem appropriate to be fully implemented onto the Malaysian data since it is fundamentally designed based on UK data. Therefore, this paper only adapts the concept of the Wilkie model with Malaysian dataset but formulating each modified asset model using a suitable Box-Jenkins model.

a) Outline of the Approach:

The fundamental variables selected for this research are based on the basic variables contained in the Wilkie model first derived in 1986 and the subsequent variables updated in Wilkie (1995) as these variables are believed to have major representative roles towards the leading asset classes categorised in Malaysia (Ishak, 2015). Relatively, the basic asset classes that involved in this study are stock and long-term security (bond). Yet, since the main goal of annuity scheme investment is to preserve the annuitants' benefits by maximising the funds in the long run, property asset would be included in the development of asset model in this paper. This is because many papers including Mugambi (2014) has argued that property investment is a tangible asset with low volatility which generates long-term capital appreciation that fulfils the goal of annuity fund. Nevertheless, this paper also studies inflation rates as well as short-term interest rate (real interest rate) as they have great impact towards an individual's lifetime financial needs. After taking all the factor described into consideration, it is concluded that all the four fundamental and two newly updated Wilkie models will be employed in this research paper. Specifically, the six variables would be categorised accordingly to business perspective and individual perspective in the next section along with explanations on the data used to model them.

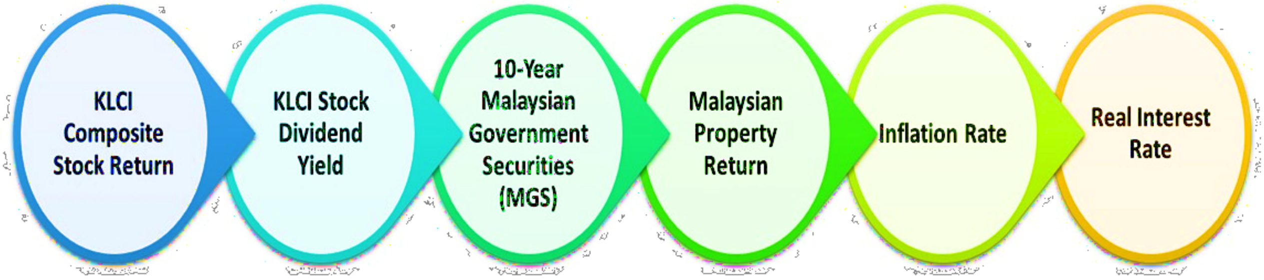


Figure 2: Summary on the six models employed

b) Fundamental Variables Modelled from Business Perspective:

KLCI Composite Stock Return

The data representing composite stock return is the FTSE Bursa Malaysia KLCI Prices Index. The composite stock return is measured as:

$$Composite\ Stock\ Return = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

(Equation 1)

where

P_t = Stock Prices index at time t,

P_{t-1} = Stock Prices index at time $t-1$.

The data was collected from Yahoo Finance governed by Malaysia Government, which available in monthly basis from 1994 to 2017. The data is available in https://finance.yahoo.com/

KLCI Stock Dividend Yield

The data representing the dividend yield is the FTSE Bursa Malaysia KLCI yield. The data was gathered by downloading each monthly report of FTSE Bursa Malaysia. Unfortunately, the data available is on monthly basis which consists of only three years from 2015 to 2017.

10-Year Malaysian Government Securities (MGS)

The data representing the bond yield is the 10-year Malaysian Government Securities (MGS) yield which reflects the long-term interest-bearing securities in Malaysia. The data was downloaded from the Department of Statistics Malaysia (www.dosm.gov.my) whereby it is only available in yearly basis, starting 2004 to 2017.

Malaysian Property Return

The data representing the property return in Malaysia is the Housing Prices Index (HPI) of Malaysia. The property return is measured as:

$$Property\ Return = \ln\left(\frac{HPI_t}{HPI_{t-1}}\right)$$

(Equation 2)

where

HPI_t = Housing Prices index at time t,

HPI_{t-1} = Housing Prices index at time $t-1$.

The data was collected from Central Bank of Malaysia (www.bnm.gov.my), which available in quarterly basis from 2000 to 2014.

c) Fundamental Variables Modelled from Individual Perspective:

KLCI Composite Stock Return

The data representing composite stock return is the FTSE Bursa Malaysia KLCI Prices Index. The composite stock return is measured as:

$$Composite\ Stock\ Return = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

(Equation 3)

where

P_t = Stock Prices index at time t,

P_{t-1} = Stock Prices index at time $t-1$.

The data was collected from Yahoo Finance (https://finance.yahoo.com/) governed by Malaysia Government, which available in monthly basis from 1994 to 2017.

KLCI Stock Dividend Yield

The data representing the dividend yield is the FTSE Bursa Malaysia KLCI yield. The data was gathered by downloading each monthly report of FTSE Bursa Malaysia (www.bursamalaysia.com). Unfortunately, the data available is on monthly basis which consists of only three years from 2015 to 2017.

10-Year Malaysian Government Securities (MGS)

The data representing the bond yield is the 10-year Malaysian Government Securities (MGS) yield which reflects the long-term interest-bearing securities in Malaysia. The data was downloaded from the Department of Statistics Malaysia (www.dosm.gov.my) whereby it is only available in yearly basis, starting from 2004 to 2017.

Inflation Rate

The data representing the inflation rate in Malaysia is the Consumer Prices Index. The composite stock return is measured as:

$$Inflation\ Rate = \ln\left(\frac{CPI_t}{CPI_{t-1}}\right)$$

(Equation 4)

where

CPI_t = Consumer Prices index at time t,

CPI_{t-1} = Consumer Prices index at time $t-1$.

The data was acquired from the Department of Statistics Malaysia (www.dosm.gov.my) whereby it is only available in yearly basis, starting from 1961 to 2016.

Real Interest Rate

The data of real interest rate which used to represent the short-term interest in Malaysia is easily assessable from the international website (TheGlobalEconomy.com, 2018). Specifically, the data available is in yearly basis, starting from 1990 to 2016.

d) Box-Jenkins Modelling:

Basically, Box-Jenkins models are the fundamental to the development of Wilkie model which was first discussed in a highly influential book written by statisticians George Box and Gwilym Jenkins in 1970 (Box and Jenkins, 1970). Commonly, Box-Jenkins modelling refers to a statistical approach to identify an appropriate ARIMA process for a stationary time-series dataset, fitting it to the data, and subsequently using the best fitted model for forecasting or simulation. Relatively, an appropriate model is chosen from its rich family of models, comprising of an autoregressive model (AR), a moving average model (MA), an autoregressive moving average model (ARMA) and a Box-Jenkins model for a non-stationary series which is an autoregressive integrated moving average model (ARIMA). As compared to other modelling approaches, Box-Jenkins approach is chosen in this research as many studies have claimed that simulation from simple ARIMA models have regularly outperformed other modelling techniques especially for Malaysian time-series data with irregular pattern (Shabri, 2001). In fact, since the investment data collected in this study is mostly more than 5 years, it is believed that Box-Jenkins modelling technique is the most practical method to be applied onto the dataset as it is proven to usually obtain the best forecasting results from historical data available at least 5 to 10 years.

e) Type of Box-Jenkins Models:

Time series models developed on the basis of the Box-Jenkins approach are generally known as the univariate Box-Jenkins models, which used to establish a relationship between the present value of a time series and its historical values so that future values could be simulated according to the past values alone (Wang, 2008). Essentially, the descriptions on the four types of Box-Jenkins models including an autoregressive model (AR), a moving average model (MA), an autoregressive moving average model (ARMA) as well as an autoregressive integrated moving average model (ARIMA) would be demonstrated in this section.

Autoregressive (AR) Model

The autoregressive model of order 1 or known as AR (1) is in the form of

$$\mathrm{y_t} = \delta + \phi_1 \mathrm{y_{t-1}} + \varepsilon_t$$

(Equation 5)

where

$\mathrm{y_t}$ = a time series,

δ = a constant,

ϕ_1= an autoregressive coefficient,

ε_t= a series of errors at time *t* with zero-mean and variance σ_ε^2.

The AR (1) model is a simple linear regression model where $\mathrm{y_t}$ denotes the dependent variable while $\mathrm{y_{t-1}}$ denotes the independent variable. By taking the expectation to the equation $\mathrm{y_t}$, it obtains the following equation:

$$E[\mathrm{y_t}] = E[\phi_1] + E[\mathrm{y_{t-1}}] + E[\varepsilon_t]$$

(Equation 6)

with $E[\varepsilon_t]$ = 0. Stationary conditions are assumed as $|\delta| < 1$ and $E[\mathrm{y_t}] = E[\mathrm{y_{t-1}}] = \mu$, it produces a result of $\mu = \delta + \phi_1\mu$. Thus, $\mu = \frac{\delta}{1-\phi_1}$ is the mean of the AR (1) model. The constant δ is related to the mean μ. This relation suggests that the mean only exists if $\phi_1 \neq 1$ and the mean is zero if and only if $\delta = 0$. Therefore, δ can be uttered as $\delta = (1 - \phi_1)\mu$.

By substituting δ into the equation $\mathrm{y_t}$, it obtains

$$\mathrm{y_t} = (1 - \phi_1)\mu + \phi_1 y_{t-1} + \varepsilon_t$$

$$y_t = \mu - \phi_1\mu + \phi_1 y_{t-1} + \varepsilon_t$$

$$\mathrm{y_t} - \mu = \phi_1(\mathrm{y_{t-1}} - \mu) + \varepsilon_t.$$

The prior equations are repeatedly substituted and achieved

$$\mathrm{y_t} - \mu = \varepsilon_t + \phi_1\varepsilon_{t-1} + \phi_1^2\varepsilon_{t-2} + \cdots = \sum_{i=0}^{\infty} \phi_1^i\, \varepsilon_{t-i}.$$

(Equation 7)

This shows that $y_t - \mu$ is linearly dependent on ε_{t-i} when $i \geq 0$. By taking the square and expectation to the equation above, the variance of this series obtained is

$$Var[y_t] = \frac{\sigma_\varepsilon^2}{1-\phi_1^2}$$

(Equation 8)

where $\phi_1^2 < 0$. Under the generalisation of the AR (1) model, the autoregressive model of order *p* or written as AR (*p*), with non-negative integer *p*, satisfies the following equation:

$$y_t = \delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t.$$

(Equation 9)

The process y_t is a linear function of the *p*-th past values of itself with some errors ε_t which states any information left by the past values. Assume that ε_t is independent of the process $y_{t-1}, y_{t-2}, \ldots, y_{t-p}$.

Moving Average (MA) Model

The MA model is a simple extension of white noise series which built by multiplying the weight 1, $-\theta_1, -\theta_2, \ldots, -\theta_q$ to error terms $\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots, \varepsilon_{t-q}$ and further, moving the weights to $\varepsilon_{t+1}, \varepsilon_t, \varepsilon_{t-1}, \ldots, \varepsilon_{t-q+1}$ to get y_{t+1} process and this concept will continue for the rest. The moving average model of order 1, MA (1) is in the form of

$$y_t = \delta + \varepsilon_t - \theta_1 \varepsilon_{t-1}$$

(Equation 10)

where

ε_{t-1} = error of the series at time $t-1$,

ε_t = error of the series at time t,

θ_1= the coefficient of the first order moving average,

δ = a constant.

By taking the variance to the equation above, the variance of this series obtained is

$$Var[y_t] = \sigma_\varepsilon^2 + \theta_1^2 \sigma_\varepsilon^2 = \sigma_\varepsilon^2 (1 + \theta_1^2)$$

(Equation 11)

with σ_ε representing a standard deviation of ε_t. From the MA (1) equation, a large value of θ_1 shows that the process y_t is influenced heavily by the previous values of the error. In fact, the moving average model of order *q* which is known as MA (*q*) is

$$y_t = \delta + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q}.$$

(Equation 12)

Cleary, the series y_t is considered to be a linear process of its past and present errors and does not depend on its past values.

Autoregressive Moving Average (ARMA) Model

The idea of the development of the ARMA model is to prevent a high number of parameters that AR or MA models may have. Therefore, the ARMA model combines the AR and MA terms into a compact form so that the number of parameters is kept small. The ARMA (*p,q*) or to be understood as ARMA model of order *p* and *q*, is in the form of

$$\mathrm{y_t} = \delta + \phi_1 \mathrm{y_{t-1}} + \cdots + \phi_p \mathrm{y_{t-p}} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \cdots - \theta_q \varepsilon_{t-q}$$

(Equation 13)

where

δ = a constant,

$\phi_1, \ldots, \phi_p$= autoregressive parameters,

$\theta_1, \ldots, \theta_q$= moving average parameters,

$\{\varepsilon_t\}$ = a series of errors

The unconditional mean of the ARMA (*p,q*) is given by

$$E[\mathrm{y_t}] = \frac{\delta}{1 - \phi_1 - \cdots - \phi_p}.$$

(Equation 14)

Autoregressive Integrated Moving Average (ARIMA) Model

Essentially, the Box-Jenkins models which mentioned earlier are only fitted the stationary series. For non-stationary series such as economic and stock price series, the suitable Box-Jenkins model to treat this kind of series is an autoregressive integrated moving average, also known as ARIMA model. The non-stationary series after the first difference can be written as the $\overline{y_t} = \mathrm{y_t} - \mathrm{y_{t-1}}$ or in another notation as $\overline{y_t} = \nabla^d \mathrm{y_t}$.

The ARIMA (p,d,q) model of order p and q with difference d is shown as the following equation:

$$\bar{\mathrm{y}}_\mathrm{t} = \phi_1 \bar{\mathrm{y}}_\mathrm{t-1} + \phi_2 \bar{\mathrm{y}}_\mathrm{t-2} + \cdots + \phi_p \bar{\mathrm{y}}_\mathrm{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q}.$$

(Equation 15)

By the substituting the first difference equation into the equation above, ARIMA (p,1,q) is obtained as

$$\mathrm{y_t} - \mathrm{y_{t-1}} = \phi_1(\mathrm{y_{t-1}} - \mathrm{y_{t-2}}) + \phi_2(\mathrm{y_{t-2}} - \mathrm{y_{t-3}}) + \cdots + \phi_p(\mathrm{y_{t-p}} - \mathrm{y_{t-P-1}}) + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q}$$

(Equation 16)

and it can be written as

$$\mathrm{y_t} = (1 + \phi_1)\mathrm{y_{t-1}} + (\phi_2 - \phi_1)\mathrm{y_{t-2}} + (\phi_3 - \phi_2)\mathrm{y_{t-3}} + \cdots + (\phi_p - \phi_{p-1})\mathrm{y_{t-p}} - \phi_p \mathrm{y_{t-p}} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q}.$$

(Equation 17)

f) Box-Jenkins Modelling Procedures:

After introducing the four types of Box-Jenkins models in previous section, this section demonstrates the modelling process of Box-Jenkins approach in conjunction to fit each time series data with the most appropriate model that able to forecast accurately based on a description of historical pattern in the data. The Box-Jenkins methodology determines the best fit model with three-step iterative approach where the process is repeated several times until a satisfactory model is finally identified. The three-step iterative routine includes model identification, parameter estimation, and diagnostic checking.

Step 1: Model identification

According to Box-Jenkins methodology, a time series data must be in its stationary state before it can be modelled. A time series is considered as stationary if its mean and variance tend to vary about the same level and the variability does not fluctuate over time (Ihaka, 2005). A simple way to determine the stationarity of a time series data is by examining the pattern of the time series values plotted in the graph. Specifically, the plot is vital to reveal important pattern of the series such as trend, seasonality as well as outliers (Ishak, 2015). Yet, besides looking at the plot to determine the stationarity of a series, the sample autocorrelation function (ACF) also gives visibility on the stationarity of a series. If the ACF of the time series value either cuts off or dies down fairly quick (Figure 3.3(a)), then the time series values should be considered stationary. However, if the ACF of the time series values either cuts off or dies down extremely slow (Figure 3.3(b)), then the time series should not be considered stationary (Hoang, 2013).

Significantly, stationarity checking is the most crucial part before a time series could proceed to the modelling steps. Therefore, instead of only justifying the stationarity of a time series according to its plot and ACF which shows no exact accuracy, an Augmented Dickey-Fuller unit root test (ADF test) is necessary to be employed in this research in order to provide a more persuasive evidence on the stationarity check of a time series. ADF test is chosen to perform the stationary check in this research as this unit root test has been widely used for testing stationarity over the past few years due to its accuracy (Baumöhl, Lyócsa and Lyócsa, 2009).

Consequently, if the series is not stationary, first or second differencing transformation is performed on the original data to produce stationary time series values. According to Adhikari and Agrawal (2013), it has argued that differencing transformation is an appropriate way of transforming the non-stationary data into a stationary form as differencing helps to stabilize the mean of series by removing changes in the level of the series as well as eliminating trend and seasonality that existed in the time series. Practically, the series would be in its stationary state at most at the second differencing. In this research, a function "ndiffs" in R is used to check how many non-seasonal differencing is needed to transform the non-stationary data into its stationary form.

For non-stationary time series data, its differentiated series after the differencing transformation follows the ARIMA model. ARIMA (p,d,q) has the same form as ARMA (p,q) except for the existence of the number of difference, d, in ARIMA. Originally, suitable provisional values for p and q are determined by analysing a plot of ACF and partial autocorrelation function (PACF) of the series. However, just by analysing through the plot, the accuracy of the provisional values determined for p and q could not be promised. Therefore, instead of choosing the suitable provisional values for p and q manually, the "auto.arima" function in R is applied in this study to better assist the researcher in defining the most suitable provisional values for p and q automatically.

Step 2: Parameter Estimation

The historical data are used to generate the values of parameters of δ, $\phi_1,\ldots,\phi_p$ and $\theta_1,\ldots,\theta_q$ in the model. In conjunction to select the most appropriate model, it is crucial to determine the optimal model parameters. Typically, there will be several ARMA/ARIMA models that could possibly fit the series. Therefore, Akaike information criterion (AIC) is used in choosing the most fitted ARMA/ARIMA model. According to Tsay (2002), AIC is a fined technique that used to select a "better" model that minimises the Kullback-Leibler distance between the model and the truth. Therefore, an ARMA/ARIMA model with the lowest AIC is chosen as it is believed to have the best fit to the truth among the other possible model. As such, since "auto.arima" function is employed in this research, it would automatically help the researcher to obtain the best fit ARIMA model with optimal parameters that gives the lowest AIC.

Step 3: Diagnostic Checking

Basically, the objective of diagnostic checking is to ensure the estimated model is adequate enough to describe the real-world situation. Regardless of what estimation procedure is employed in modelling, the criteria for testing the goodness of fit are always similar. In this step, the adequacy of the estimated model is checked and if needed, an improved model would be suggested. The appropriate way to check the adequacy of an overall Box-Jenkins model is to examine its residuals. The residuals are calculated as the difference between the actual values and the fitted values whereby the residuals are unpredictable in every observation. There are few criteria to look into in order to check the adequacy of estimated model.

Firstly, the plot of residuals must show no pattern as the existence of pattern indicates the non-linear relationship between error terms. If the errors are not independent from each other, the estimate of the error standard deviation will be biased, potentially leading to improper inferences about the process. Secondly, there should be no serial correlation between residuals whereby the autocorrelations at all lags should be nearly zero, so that the residuals are proven to follow a white noise process. In addition, the autocorrelations must all be within the 95% zero-bound to show that the residuals are uncorrelated. If there are correlations between residuals, it implies that there is still information left in the residuals which have not been fully utilised in computing the forecasting. Thirdly, the Ljung-Box test is commonly used in ARIMA modelling to test the overall randomness based on the number of lags by applying to the residuals of a fitted ARIMA model (Ljung and Box, 1978). However, there is very little practical guideline on how to choose the number of lags for the test. As such, this paper follows the guideline demonstrated in the book, (Hyndman and Athanasopoulos, 2014) whereby the appropriate number of lags that used to test the randomness of non-seasonal data is suggested to be at least 10 lags. In fact, by examining the plot of Ljung-Box p-values of fitted model, if the p-values for each lag is greater than 0.05, then the residuals are proven to be independent whereby the fitted model is adequate. If the chosen model is inadequate, then the model would be reformulated.

g) Simulation:

By precisely following the three demonstrated Box-Jenkins modelling steps, the final systems for all the six fundamental variables are achieved. For each final model constructed, it is used to simulate the future time series values for next 35 years up to 10,000 times. Specifically, R software is chosen to perform the simulation as this software is relatively easy to use Rakshan and Pishro-Nik (2017) and it generates more appropriate and precise estimation. Significantly, the point of prediction of

$$y_t = \delta + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \cdots - \theta_q \varepsilon_{t-q}$$

is

$$\hat{y}_t = \delta + \hat{\phi}_1 y_{t-1} + \cdots + \hat{\phi}_p y_{t-p} + \hat{\varepsilon}_t - \hat{\theta}_1 \hat{\varepsilon}_{t-1} - \cdots - \hat{\theta}_q \hat{\varepsilon}_{t-q}$$

$$\hat{\varepsilon}_t \sim iid(0, \sigma^2).$$

(Equation 18)

3. Results and Discussion

3.1 Optimal Fund Allocation from Business Perspective

Within this context, the projected liability cash flows are combined with the asset cash flows to obtain the probability whereby the asset cash flows are more than the liability cash flows by substituting each weightage combinations into the asset cash flows. From here, the optimal fund allocation which maximises the investment return of annuity insurers could be obtained from the weightages combination that gives the highest probability of solvency. Specifically, the top-10 weightages combinations which return the top-10 highest probability of solvency level for the single premium invested of RM100,000 are demonstrated in the table as shown below:

No.	W(S)	W(B)	W(P)	Probability
135	0.05	0.94	0.01	0.967
159	0.06	0.93	0.01	0.9662
57	0.02	0.97	0.01	0.966
110	0.04	0.95	0.01	0.966
134	0.05	0.93	0.02	0.966
158	0.06	0.92	0.02	0.9659
182	0.07	0.92	0.01	0.9658
109	0.04	0.94	0.02	0.9656
84	0.03	0.96	0.01	0.9655
29	0.01	0.98	0.01	0.9654

Figure 3: The Top-10 Optimal Weightage Combinations

Basically, by observing through the table above, it implies that the most optimal weightages combination which gives the highest probability of solvency level among the others is the weightages combination at position 135.

Specifically, the optimal weightages combination suggests that the annuity insurers should allocate 94% of the single premium, RM100,000 which collected from the annuitant based on the early assumption in this research to the bond market, whereas the other 5% of the fund should be invested in stock but only 1% of the premium should be allocated into the property investment. By having most of the fund allocated in Malaysian bond market along with lesser investments into stock and property market, it ensures a solvency level of above 95% for the annuity fund throughout the projected period. Significantly, it implies that this result is in line with Battocchio, Menoncin and Scaillet (2003) which argued that the investment in the risky assets especially stocks should be decreased in order for annuity insurers to meet their long-term future contractual annuity payments to the annuitants. In addition, another evidence to support this result came from the research carried out by Bagliano, Fugazza and Nicodano (2009) who suggested that annuity insurers should hold a larger fraction of their portfolio in bonds which commonly known as the safest investment instrument that could preserve the fund value in long run to meet their long-term liabilities. In fact, the result shown in this research appeared to be consistent with the optimal asset allocation policy applied to the government pension funds - Employees' Provident Fund (EPF) which have been established by the EPF committee boards. According to the quantitative regulation, the government pension funds in Malaysia is required to keep 70% of its assets into Malaysian government securities and only allowed to invest up to 25% of its assets in risky assets (Ghosh, 2006). Significantly as shown in Figure 3, the optimal fund allocation discovered in this research has met the general optimal asset allocation rules in Malaysia whereby it has suggested that more than 70% of the fund should invest in bonds while less than 25% of fund should be allocated in stock.

On the other hand, a significant finding to differentiate this paper from other research is that this study has suggested to invest almost the whole proportion of the annuity fund into the bond market due to the more appealing bond market predicted over the next 35 years. Originally, investment in bonds usually encounters lower risk but with less returns as compared to the risky stock investments which subjected to higher expected returns. However, as discussed earlier, the predicted bond market over the next 35 years does not show in this way. Specifically, the simulated result on the future Malaysian bond market suggests that the Malaysian bond market over the next 35 years is predicted be less volatile and at the same time surprisingly providing higher opportunity for investors to gain more returns compared to the stocks. Therefore, instead of investing in stock market which appears to be riskier and subjected to lower returns, it is very logical for annuity insurers to allocate more proportion of the fund into the Malaysian bond market over the next 35 years which is proven to be less volatile but with greater opportunity to maximise return.

Although the stock market over the next 35 years is predicted to be more volatile and associated with lower return, the overall expected total stock return generated is still higher than the expected property return predicted over the next 35 years. Therefore, more fund is suggested to be invested in stock market instead of property market. In fact, even though the Malaysian property market is predicted to be more stable and associated with higher earnings potential over the next 35 years as compared to its historical years, the increase in its potential earnings still not as high as the potential returns expected to generate from bond market. Hence, in conjunction to maximise investment returns, it is no doubt that more funds should be invested in future bond market which not only risk-free but also provides higher earnings potential.

3.2 Premiums Simulated with Optimal Fund Allocation Weightages

Essentially, a range of premiums is simulated with the optimal fund allocation weightages defined in previous section through R in order to identify the minimum premium that should be subjected to annuitants in conjunction to at least maintain the solvency level of the insurance companies at 90%. As such, the simulated result is shown as follows:

No.	W(S)	W(B)	W(P)	Probability	Premium
1	0.05	0.94	0.01	0.7924	10000
2	0.05	0.94	0.01	0.9171	20000
3	0.05	0.94	0.01	0.9423	30000
4	0.05	0.94	0.01	0.9519	40000
5	0.05	0.94	0.01	0.9566	50000
6	0.05	0.94	0.01	0.9601	60000
7	0.05	0.94	0.01	0.9626	70000
8	0.05	0.94	0.01	0.964	80000
9	0.05	0.94	0.01	0.9658	90000
10	0.05	0.94	0.01	0.967	100000
11	0.05	0.94	0.01	0.9678	110000
12	0.05	0.94	0.01	0.9682	120000
13	0.05	0.94	0.01	0.9687	130000
14	0.05	0.94	0.01	0.9691	140000
15	0.05	0.94	0.01	0.9695	150000
16	0.05	0.94	0.01	0.9696	160000
17	0.05	0.94	0.01	0.9697	170000
18	0.05	0.94	0.01	0.9699	180000
19	0.05	0.94	0.01	0.97	190000
20	0.05	0.94	0.01	0.97	200000
21	0.05	0.94	0.01	0.9703	210000
22	0.05	0.94	0.01	0.9705	220000
23	0.05	0.94	0.01	0.9707	230000
24	0.05	0.94	0.01	0.9707	240000
25	0.05	0.94	0.01	0.971	250000
26	0.05	0.94	0.01	0.9711	260000
27	0.05	0.94	0.01	0.9712	270000
28	0.05	0.94	0.01	0.9713	280000
29	0.05	0.94	0.01	0.9713	290000
30	0.05	0.94	0.01	0.9713	300000
31	0.05	0.94	0.01	0.9714	310000
32	0.05	0.94	0.01	0.9715	320000
33	0.05	0.94	0.01	0.9716	330000
34	0.05	0.94	0.01	0.9718	340000
35	0.05	0.94	0.01	0.9719	350000
36	0.05	0.94	0.01	0.9719	360000
37	0.05	0.94	0.01	0.972	370000
38	0.05	0.94	0.01	0.972	380000
39	0.05	0.94	0.01	0.972	390000
40	0.05	0.94	0.01	0.972	400000
41	0.05	0.94	0.01	0.972	410000
42	0.05	0.94	0.01	0.972	420000
43	0.05	0.94	0.01	0.972	430000
44	0.05	0.94	0.01	0.972	440000
45	0.05	0.94	0.01	0.9722	450000
46	0.05	0.94	0.01	0.9722	460000
47	0.05	0.94	0.01	0.9722	470000
48	0.05	0.94	0.01	0.9722	480000
49	0.05	0.94	0.01	0.9722	490000
50	0.05	0.94	0.01	0.9722	500000
51	0.05	0.94	0.01	0.9723	510000

Figure 4: The Range of Premiums Simulated with the Optimal Fund Allocation Weightages

By simulating a list of premiums with the same optimal fund allocation weightages, the minimum premium which required to ensure the solvency level of the insurance companies is at least maintained at 90% is now obtained. Referring to Figure 4, it shows that a single premium of RM20,000 is the minimum premium that should be subjected to the annuitants in order to maintain the solvency level of insurance company at 91.71%. Apparently, this implies that the minimum amount of premium subjected to the annuitants could be one of the factors that explains the fragile and unstable annuity funds in Asian countries. For example, the minimum premium subjected to the annuitants of PRUretirement Growth Plan in Prudential Malaysia is as low as RM10,000.

As such, it implies that the annuity fund of PRUretirement Growth Plan may face the problem of insolvency if most of its annuitants are paying the minimum premium of RM10,000. In overall, since the ALM model in this paper is constructed with the average assumptions made based on the common characteristics shared by all annuity products available in Malaysia market, the simulated minimum premium which should be collected from annuitants to maintain the solvency level of the annuity fund at 90% and above in this paper could acts as a guideline for annuity policyholders in designing and pricing the annuity products.

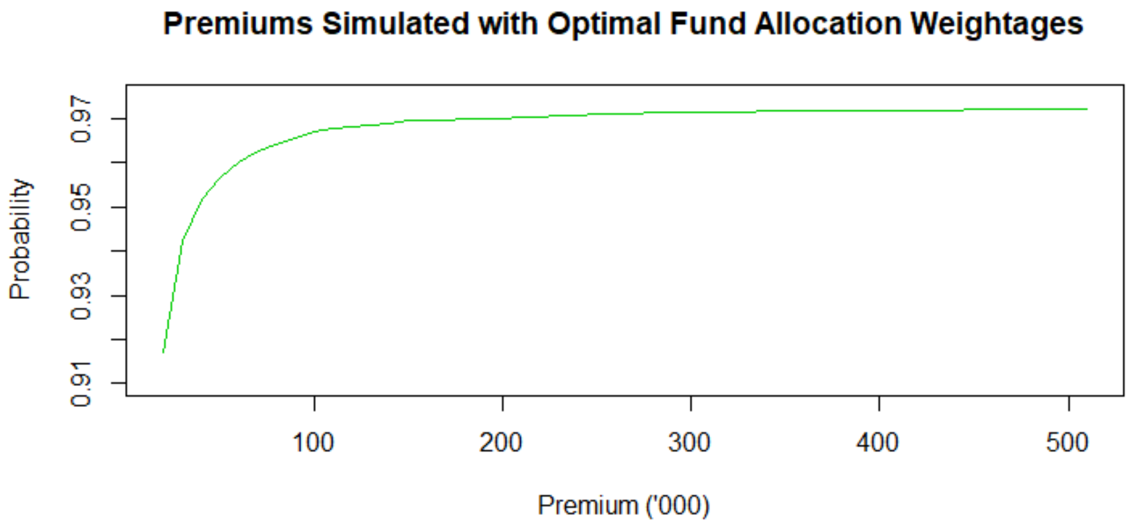


Figure 5: The Graph of Premiums simulated with the Optimal Fund Allocation Weightages

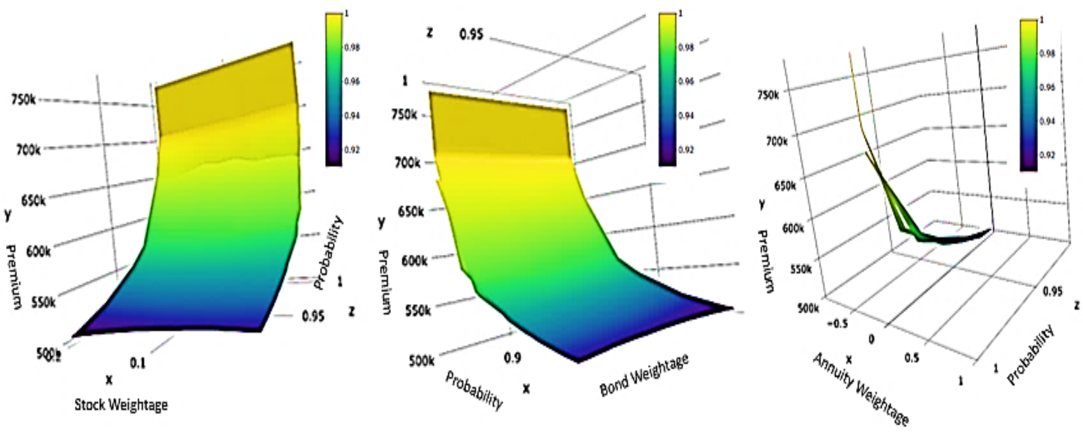
Subsequently, the range of premiums which subject to the solvency probability of more than 90% is presented in the graph as shown in Figure 4.5. Based on the graph above, it shows that as premium increases, the probability of annuity fund's solvency level also increases as more funds are available to be invested in maximising investment returns. However, to a point of time, the solvency level of annuity fund started to increase in a slower rate and eventually become constant as more increase in premiums is added. This phenomenon could be clearly explained by adapting the Law of Diminishing Marginal Utility. Based on the law, it suggests that the optimal solvency margin has been reached when more and more units of increase in premium is added to fund, the marginal utility obtained from the subsequent units of increase in premium goes on diminishing constantly (Beattie and Lafrance, 2005). Therefore, it is no point for annuity insurers to increase the premium beyond RM510,000, as the solvency level would not increase further no matter how much more unit of increase in premiums is added. Specifically displayed by the graph, the solvency margin in this case is optimised at 97.23%.

3.3 Optimal Fund Allocation from Individual Perspective

Within this context, the total accumulated asset cash flows at the age of 60 are matched with the total discounted lifetime consumptions at the age of 60 in order to acquire the probability in which the net cash flows are equal or more than zero by substituting each weightages combination along with estimated fund available for investments. By doing so, it gives a list of optimal weightages combinations in maximising the assets' returns for a range of fund available. From here, each optimal fund allocation weightages suggested for individuals with different fund available for investments within a range whereby it promises at least 90% of solvency has been demonstrated as follows:

W(S)	W(B)	W(A)	Probability	Fund Available
0.2	0.79	0.01	1	755000
0.18	0.81	0.01	1	760000
0.15	0.84	0.01	1	765000
0.13	0.86	0.01	1	770000
0.11	0.88	0.01	1	775000
0.09	0.9	0.01	0.9997	705000
0.08	0.91	0.01	0.986	585000
0.07	0.92	0.01	0.9573	535000
0.06	0.93	0.01	0.9473	525000
0.05	0.94	0.01	0.9222	505000
0.04	0.95	0.01	0.9287	510000
0.03	0.96	0.01	0.9418	520000
0.02	0.97	0.01	0.9756	560000
0.01	0.98	0.01	0.9994	680000

Figure 6: The Optimal Fund Allocation Weightages Suggested for Each Fund Available



(a) Stock Asset (b) Bond Asset (c) Annuity Asset

Figure 7: The Optimal Stock, Bond and Annuity Allocation for Each Fund Available

Basically, Figure 6 shows the optimal fund allocation weightages suggested for a fund available within a range of RM505,000 to RM775,000 with the solvency margin maintained above 90%. Significantly, this result could act as a guideline for individuals with different capitals available for investments to allocate their fund optimally in each asset class. Overall, the result shown in the table above suggests that an optimal investment portfolio for retirees should consists of both non-annuitized assets and annuity product at which most of their fund should be allocated in bond, followed by stock and annuity. As such, this finding is found consistent with Horneff et al. (2007) who claimed that an individual could take advantage on the diversification effect by investing into non-annuitized and annuitized products which significantly lower down the portfolio risk. To the great extent, the findings in this research which suggest investors to allocate part of their investment fund into annuity products has shed a light on the importance of annuity products in protecting the retirees against longevity risk. Significantly, it implies that annuity products play a part in making retirees better off.

However, the findings in this research also suggests that investors should invest the least in annuity products as compared to bond and stock. This may due to the fact that although annuity products could give support to retirees in sustaining their retirement life by guaranteed them a steam of income after they retire, the guaranteed income payable to retirees is obviously not substantial enough to fully cover their lifetime consumption. In fact, the income return generated by annuity products are much lesser compared to bond as annuity product is considered an insurance policy with primary goal of giving protection rather than serving as an investment instrument that maximise returns. On the other hand, investors who are soon approaching their retirement age are generally more risk- adverse. Therefore, they are advised to allocate more fund in bonds which are less uncertain compared to stocks. In fact, as mentioned before, the simulated bond return model has come out a prediction that the Malaysian bond market in the next 35 years not only incurs lower risk but at the same time potentially providing higher opportunity for investors to gain higher return than investing in stocks. Therefore, the optimal fund allocation weightages for bond is generally much higher compared to the stock as shown in the table above.

By referring to Figure 6 again, there actually exists a pattern in the optimal fund allocation weightages for each asset class. Specifically, the relationship between the optimal weightages for each asset and their corresponding fund required as well as the solvency probability is well displayed in a 3D-graph as shown in Figure 7. Relatively, the inverse relationship between the weightage of fund allocated in stock and the weightage of fund allocated in bond is clearly displayed in Figure 3.32. Given that the allocation rate of fund in annuity products is always constant, the weightage of fund allocated in stock tend to decrease when more funds are invested in bond asset.

Initially, when the weightage of fund allocated in stock decreases from 20% to 11%, the total fund required for investment increases from RM755,000 to RM775,000 while the solvency level is remained constant at 100%. Similar to this, when the weightage of fund allocated in stock decreases from 4% to 1%, the total available fund needed for investment tends to rise from RM505,000 to RM680,000 with respect to the increase in its solvency probability. From here, it implies that if investors are more risk-adverse and would not want to invest more in stocks, they have to ensure that their total capital available for investment are large enough. Fundamentally, stocks are short-term investment with money being tied up for less than a year while bonds are usually referred to a long-term investment where money is typically tied up for at least 10 years. As such, it is not worth for investors to only invest a small amount of fund into the 10-year MGS bonds with money being tied up for 10 years but could not generate any huge interest due to the limited capitals invested. Hence, if investors prefer to allocate more fund in bonds, they have to ensure that the capital available for them to do this long-term investment is big enough to earn them a larger amount of interest in long run.

However, in exceptional cases occur when the weightage of fund allocated in stock falls from 9% to 5%, the funds needed for investment also decreases from RM705,000 to RM505,000. On the other hand, Figure 3.32(c) shows that the weightage for fund allocated in annuity products is always constant at 1%. From here, it implies that only 1% of fund allocated in life annuity products is already enough for investors to enjoy the maximum protection from annuity product. Since investors are concerned in maximising their return to cover their lifetime consumption during retirement, they should focus more on the investment instruments that give higher potential returns such as stocks and bonds, but not the annuity products which served as an insurance which give protection but not optimal returns.

4. Conclusion

Empirically, this research has conducted a profound evaluation into the annuity market in Malaysia from both the business and individual perspective. To the great extent, analysis is made on how the annuity insurers could better maintain the solvency level of annuity fund which exposed under significant economic risks by obtaining an optimal fund allocation to maximise investment returns through the development of ALM. Along with this, valuation from individual perspective is made by constructing the ALM to identify the optimal investment strategy for investing in both the non-annuitized and annuity products which give adequate returns to cover the lifetime consumptions of an individual when he or she retirees. Specifically, the specific objectives demonstrated in the beginning of this study has all been met.

Generally, by adapting the concept from Wilkie model, six fundamental models including inflation rate, real interest rate, KLCI stock return, KLCI stock dividend yield, 10-year MGS return as well as Malaysian property return were formulated accordingly to the Box-Jenkins modelling approach. The best fit model is determined for each of the fundamental variables following the three-step iterative approach including model identification, parameter estimation and diagnostic checking. Precisely, the six designed models have passed through all the diagnostic tests which implies the adequacy and validity of models in explaining the real-world situation. After 10,000 simulations are performed based on each constructed model, the mean and standard deviation of the simulated results appeared to be close to the mean and standard deviation of the corresponding historical values. Thus, as stated by the Law of Large Numbers, the simulated results have high accuracy and reliability as they are large enough to trace the characteristics of the historical data to predict the actual movement of the variables over the next 35 years.

According to the simulated results, the inflation rate in Malaysia over the next 36 years is predicted to persist at a lower but stable rate, whereby a sustainable growth in Malaysian economy which driven by a lower consumer price should be expected. In fact, this finding is in line with the simulated real interest rate which is predicted to increase in the future. When real interest rate increases, saving become more attractive than spending, thus consumer spending decreases which eventually leads to a decrease in inflation rate too. In addition, the stock market over the next 35 years is predicted to be slightly more volatile associated with a small increase in the future dividend yield which is needed to compensate the extra risk taken by the investors to continue enrolling into the stock market with higher risk and uncertainty. However, in contrast to the high volatility in stock market, the forecasted bond market within next 35 years not only appears to be less volatile but at the same time potentially providing investors with higher chance to gain more yield, same goes to the property market.

After the ALM is constructed from business perspective, the optimal fund allocation which returns the highest probability of solvency level for annuity fund of RM100,000 is obtained. Specifically, the result suggests that 94% of the annuity fund should be put into in the bond asset whereas the other 5% of the fund should be invested in the stock and only 1% of the fund should be allocated in property market in order to maximise the investment return. From here, it implies that bonds are safer instrument to invest in compared to stocks and property as they are associated with lower risk which make them more suitable for investors with higher level of risk-aversion especially retires who cannot afford to lose any of their savings. In fact, by simulating a range of premiums with the optimal fund allocation weightages, the result found out that the RM20,000 is the minimum premium that should be subjected to annuitants in conjunction to at least maintain the solvency level of the annuity fund at 90%. Thus, this could act as a guideline for annuity policyholders in pricing the annuity products.

In term of individual investment, this research has suggested that the decision to allocate certain percentage of fund in stocks and bonds is very subjective and it varies among people since the decision depends largely on investors' risk appetites as well as their living lifestyle. Certainly in this research, it has recommended that the optimal weightage of fund to be invested in stock is varying from 1% to 20% while the optimal fund allocation weightages in bonds are ranged from 79% to 98%, given the available fund size within RM505,000 to RM755,000. In fact, this research has further hinted that an individual should at most allocate 1% of fund in life annuity products since annuities are served as protection instruments which do not generate high investment returns. Therefore, if investors are concerned in maximising their return to cover their lifetime consumption during retirement, it explains the result in this paper for suggesting a higher weightage of fund to be allocated in stocks and bonds which serves as investment instruments that give higher potential returns. In overall, the optimal fund allocation strategy for both annuity insurers and individuals suggest a higher allocation of funds in bond. Thus, it concludes that bond asset plays an important role in maximising the annuity fund returns to prevent deficit of scheme whereby annuitants are all at risk of receiving only reduced retirement income.

4.1 Recommendations

Essentially, this study is only focused on Malaysian content at which the ALM constructed is for Malaysian annuity fund as well as the individuals who are soon enough to reach their retirement age in this country.

Relatively, all assumption made in developing the ALM are based on the current annuity products available in Malaysia. As such, the results are concluded according to the data collected in Malaysia which could not be used as a comparison as there are limited published papers focusing on the development of ALM for Malaysian annuity fund. Therefore, the recommendation for future researchers is that they should conduct further study deep into the annuity market in Malaysia so that more results could be used as comparison to better reflect the actual situation of annuity market in Malaysia. In fact, they could also examine the determinants on the demand of life annuity products in Malaysia which has not yet been discovered by any researchers.

On the other hand, the findings in this research could benefit the annuity insurers and investors in assisting them to make wise investment decisions. Since the optimal fund allocation strategy for annuity fund is proposed in this research, it is believed that it could act as a guideline for annuity insurers to construct a better investment portfolio plan that provide them with maximum returns to ensure their asset cash flows are always adequate enough to meet their long-term liabilities payable to annuitants. Collectively, the results in this paper provides individuals with early preparation before they reach their retirement age by revealing an optimal fund allocation investing in the combination of non-annuitized assets and annuity product which could maximise their retirement income in compensating their lifetime financial needs.

According to the optimal fund allocation strategy proposed to individuals in this research, it suggests that investors should invest the least in annuity products as compared to bond and stock. Specifically, only 1% of the fund invested in the annuity products is adequate for investors to enjoy the maximum protection from annuity products. Since the aim of investors is to maximise their income, they are recommended to focus more on investment vehicle that gives higher potential returns such as bonds and stocks but not the conservative insurance policy that gives protection but with lower returns. Significantly, both the optimal fund allocation strategies suggested to annuity insurers and investors have met the general optimal asset allocation rules in Malaysia whereby more than 70% of fund is suggested to be invested in bond while not more than 25% of fund should be allocated in stock. Thus, it argues that the suggested asset allocation strategies have taken into account the typical asset allocation commonly practiced by the government pension scheme as well as Malaysian insurance companies, making it more easily to be acceptable by portfolio managers, pension insurers and board members.

Besides, the findings in this paper could also act as guideline to annuity policymakers or insurers in designing a more sophisticated annuity product which does not limited to only provide pure protection. Significantly, an annuity product which provides protection and appealing guaranteed income should be developed to better suit the needs of annuitants in Malaysia. Apart from that, this study also suggests that the minimum premium required to maintain the solvency margin of annuity fund at above 90% is RM20,000. Therefore, annuity policymakers are recommended to take this into account when pricing an annuity product so that they would not misprice an annuity policy which put the annuity fund into the danger of insolvency.

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